

CASIMIR EFFECT : THEORY AND EXPERIMENTS

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The Casimir effect is a crucial prediction of Quantum Field Theory which has fascinating connections with open questions in fundamental physics. The ideal formula written by Casimir does not describe real experiments and it has to be generalized by taking into account the effects of imperfect reflection, thermal fluctuations, geometry as well as the corrections coming from surface physics. We discuss these developments in Casimir physics and give the current status in the comparison between theory and experiment after years of improvements in measurements as well as theory.

I. INTRODUCTION

The Casimir effect [1] deserves careful attention as a crucial prediction of Quantum Field Theory [2–5]. It also has fascinating interfaces with other open questions in fundamental physics.

It is connected with the puzzles of gravitational physics through the problem of vacuum energy [6, 7] as well as with the principle of relativity of motion through the dynamical Casimir-like effects [8, 9]. Effects beyond the Proximity Force Approximation also make apparent the rich interplay of vacuum and geometry [10].

Casimir physics also plays an important role in the tests of gravity at sub-millimeter ranges [11, 12]. For scales of the order of the micrometer, gravity tests are performed by comparing Casimir force measurements with theory [13–15]. Other constraints can be obtained with atomic or nuclear force measurements [16].

Finally, the Casimir force and closely related Van der Waals force have strong connections with various active domains and interfaces of physics, such as atomic and molecular physics, condensed matter and surface physics, chemical and biological physics, micro- and nano-technology [17].

II. THE PUZZLE OF VACUUM ENERGY

The first quantum law was written by Planck in 1900 to explain the properties of the black body radiation [18]. In modern terms, it gives the mean energy per electromagnetic mode as the product $\bar{n}\hbar\omega$ of the photon energy $\hbar\omega$ by the mean number of photons per mode $\bar{n} = \left(\exp \frac{\hbar\omega}{k_B T} - 1\right)^{-1}$.

In 1911, Planck [19] wrote a new expression for the mean energy per mode $(\bar{n} + \frac{1}{2})\hbar\omega$ which contained a zero-point energy $\frac{1}{2}\hbar\omega$ besides the black body energy. In contrast to the latter, the zero-point fluctuations were still present at zero temperature. The arguments thus used by Planck cannot be considered as consistent today. The first known argument still acceptable today was proposed by Einstein and Stern [20] in 1913 : the modified Planck law reproduces the classical limit at high temperatures $(\bar{n} + \frac{1}{2})\hbar\omega = k_B T + O(\frac{1}{T})$.

In 1916, Nernst discussed zero-point fluctuations for the electromagnetic field and discovered that their energy constituted a challenge for gravitation theory [21]. When summing up the zero-point energies over all field modes, a finite energy density is obtained for the first Planck law (this is the solution of the ‘ultraviolet catastrophe’) but an infinite value is produced from the second law. When introducing a high frequency cutoff, the calculated energy density remains finite but it is still much larger than the mean energy observed in the world around us through gravitational phenomena [22].

This major problem has led famous physicists to deny the reality of vacuum fluctuations. In particular, Pauli stated in his textbook on Wave Mechanics [23] : *At this point it should be noted that it is more consistent here, in contrast to the material oscillator, not to introduce a zero-point energy of $\frac{1}{2}\hbar\omega$ per degree of freedom. For, on the one hand, the latter would give rise to an infinitely large energy per unit volume due to the infinite number of degrees of freedom, on the other hand, it would be principally unobservable since nor can it be emitted, absorbed or scattered and hence, cannot be contained within walls and, as is evident from experience, neither does it produce any gravitational field.*

A part of these statements is simply unescapable : it is just a matter of evidence that the mean value of vacuum energy does not contribute to gravitation as an ordinary energy. But it is certainly not possible to uphold that vacuum fluctuations have no observable effects. Certainly, vacuum fluctuations are *scattered* by matter, as shown by their numerous effects in atomic [24] and subatomic [25] physics. And the Casimir effect is nothing but the evidence of vacuum fluctuations making their existence manifest when being *contained within walls*.

III. THE CASIMIR FORCE

Casimir calculated the force in an idealized case, with perfectly smooth, flat and parallel plates in the limit of zero temperature and perfect reflection. The expressions for the energy $E_{\text{Cas}} = -\hbar c \pi^2 A / 720 L^3$ and force $F_{\text{Cas}} = -dE_{\text{Cas}}/dL$ thus reveal a universal behavior resulting from the confinement of vacuum fluctuations (L is the distance, A the area, c the speed of light and \hbar the Planck constant).

For the metallic mirrors used in experiments, the force depends on the optical properties of the mirror described by a dielectric function [26]. The dielectric function is a sum of contributions corresponding to interband transitions and conduction electrons. The latter contribution is directly related to the frequency dependent conductivity of the metal $\sigma[\omega] = \omega_p^2 / (\gamma - i\omega)$ where ω_p is the plasma frequency and γ the Drude damping constant. As γ is much smaller than ω_p for a good metal such as Gold, the limiting lossless case ($\gamma \rightarrow 0$) captures a large part of the effect of imperfect reflection. However it is not an accurate description: a much better fit of tabulated optical data is obtained with a non null value of γ ; moreover, a dissipative Drude model ($\gamma \neq 0$) meets the well-known fact that Gold has a finite static conductivity $\sigma_0 = \omega_p^2 / \gamma$.

Experiments are performed at room temperature so that the effect of thermal fluctuations has to be added to that of vacuum fields [27–30]. Boström and Sernelius were the first to remark that, despite its small value, γ had a large effect on the force at non null temperatures [31]. In particular, the ratio between the predictions calculated for the lossless and lossy cases reaches a factor 2 at large distances. A large number of contradictory papers has been devoted to this topic (see references in [32–35]) and the contradiction is deeply connected to the comparison between theory and experiments discussed below. It is also worth recalling here that derivations from microscopic models of the metallic mirrors give, as should be expected, Casimir forces agreeing at large distance with predictions of the dissipative Drude model [36–38].

Another important feature of the recent precise experiments is that they are performed in the plane-sphere geometry. The estimation of the force in this geometry uses the so-called *Proximity Force Approximation* (PFA) [39] which amounts to integrate over the distribution of local inter-plate distances the pressure calculated in the geometry with two parallel planes. This approximation can only be valid as a limit for sphere radius R much larger than the separation L . Even in this case its accuracy remains a question of importance for the comparison between theory and experiments discussed in the sequel of this paper. This question has been studied in recent papers [40–44] and the related advances are presented elsewhere in this volume [45].

IV. THE SCATTERING APPROACH TO THE CASIMIR EFFECT

For preparing forthcoming discussions, it is worth surveying the scattering approach which is the best tool available today for calculating the Casimir effect [46].

The basic idea is that mirrors are described by their scattering amplitudes. It can be simply illustrated with the model of scalar fields propagating along the two directions on a line (1-dimensional space; see references in [8]). Each mirror is described by a scattering matrix containing reflection and transmission amplitudes. Two mirrors form a Fabry-Perot cavity described by a scattering matrix S which can be deduced from the two elementary matrices. The Casimir force then results from the difference of radiation pressures exerted onto the inner and outer sides of the mirrors by the vacuum field fluctuations [47]. Equivalently, the Casimir free energy can be written as the sum of the frequency shifts of all vacuum field modes due to the presence of the cavity [47].

The same discussion can be extended to the geometry of two plane and parallel mirrors aligned along the axis x and y , described by specular reflection and transmission amplitudes which depend on the frequency, transverse wavevector and polarization [47]. A few points have to be treated with care when extending the derivation from 1-dimensional space to 3-dimensional space : evanescent waves contribute besides ordinary modes freely propagating outside and inside the cavity; dissipation has to be accounted for [48]. The properties of the evanescent waves are described through an analytical continuation of those of ordinary ones, using the well defined analytic behavior of the scattering amplitudes. At the end of this derivation, this analytic properties are also used to perform a Wick rotation from real to imaginary frequencies. This leads to an expression for the Casimir free energy under the form of a Matsubara formula [15].

This formula reproduces the Casimir ideal formula in the limits of perfect reflection $r \rightarrow 1$ and null temperature $T \rightarrow 0$. But it is valid and regular at thermal equilibrium at any temperature and for any optical model of mirrors obeying causality and high frequency transparency properties. It can thus be used for calculating the Casimir force between arbitrary mirrors, as soon as the reflection amplitudes are specified. These amplitudes are commonly deduced from models of mirrors, the simplest of which is the well-known Lifshitz model [49, 50]. In this model, semi-infinite bulk mirrors are characterized by a linear and local dielectric response function and reflection amplitudes are then deduced from the Fresnel law. It is worth emphasizing that the scattering formula allows to accommodate more

general expressions for the reflection amplitudes. In particular, it may be used even when the optical response of the mirrors can no longer be described by a local dielectric response function. The reflection amplitudes can for example be determined from microscopic models of mirrors. Recent attempts in this direction can be found for example in [51–55].

V. CASIMIR EXPERIMENTS

We now discuss the status of Casimir experiments and their comparison with theory. At this point, we face the difficulties that there are persisting differences between experimental results and theoretical predictions drawn from the best motivated models, as well as disagreements between some recent experiments.

On one hand, there have been experiments at IUPUI and UCR for approximately ten years, with results pointing to an unexpected conclusion [56–58]. In particular, the Purdue experiment uses dynamic measurements of the resonance frequency of a microresonator. The shift of the resonance gives the gradient of the Casimir force in the plane-sphere geometry, which is proportional (within PFA) to the Casimir pressure between two planes. The typical radius of the sphere is $R = 150\mu\text{m}$ and the range of distances $L = 0.16 - 0.75\mu\text{m}$. The results appear to fit predictions obtained from the lossless plasma model $\gamma = 0$ rather than those corresponding to the expected dissipative Drude model $\gamma \neq 0$ (see Fig.1 in [57]), in contradiction with the fact that Gold has a finite static conductivity. IUPUI and UCR experiments are performed at distances where the thermal contribution as well as the effect of γ are not large, so that the estimation of accuracy is a critical issue.

On the other hand, a new experiment in Yale [59] uses a much larger sphere $R = 156\text{mm}$, which allows for measurements at larger distances $L = 0.7 - 7\mu\text{m}$. The thermal contribution is large there and the difference between the predictions at $\gamma = 0$ and $\gamma \neq 0$ is significant. The results of this experiment see the thermal effect and they fit the predictions drawn from the dissipative Drude model, after the contribution of the electrostatic patch effect has been subtracted [59, 60]. Of course, these new results have to be confirmed by further studies [61]. The main issue in this experiment is that the pressure due to electrostatic patches is larger than that due to Casimir effect, and that the patch distribution is not characterized independently. This is in fact a more general problem since the patch distribution is not measured in other experiments either (more discussions below).

In this short discussion, we have focused our attention on a few experiments. For completeness, we give here a list of other Casimir measurements between metallic plates which have produced information of interest on the topics discussed in this paper [62–70].

VI. DISCUSSION

The conclusion at this point is that the Casimir effect, now measured in several experiments, is not yet tested at the 1% level, as has been sometimes claimed. While the Yale experiment fits predictions drawn from the dissipative Drude model, IUPUI and UCR experiments favor theoretical predictions obtained with the lossless plasma model. When comparing the IUPUI experimental data with the predictions drawn from the best motivated model (the dissipative Drude model), the pressure difference goes up to $\sim 50\text{mPa}$ at the smallest distances $\sim 160\text{nm}$ where the pressure itself is $\sim 1000\text{mPa}$. This difference is clearly larger than the statistical dispersion (see Fig.1 in [57]).

This question is important not only for the test of the Casimir effect, a central prediction of Quantum Field Theory, but also because Casimir experiments are one of the main routes in the search for hypothetical new short-range forces beyond the standard model [11, 12, 16]. The difference between IUPUI experimental data and theoretical predictions (using the Drude model) does not look like a Yukawa force law, but it looks like a superposition of power laws which meet predictions of some currently considered unification models [16].

This discrepancy between theory and experiment may have various origins, in particular artefacts in the experiments or inaccuracies in the calculations. They may also come from yet unmastered systematic effects in the analysis of experimental data. In particular recent publications study the effects of surface physics on Casimir experiments. Electrostatic patches, already alluded to above, are a probable source of such systematic effects (more discussions below). The problem of surface roughness has also to be studied in a thorough manner [71–75].

In the sequel of this paper, we discuss the effect of electrostatic patches which is a known limitation for a large number of high precision measurements [76–85], and is in particular for Casimir experiments [86–91]. The patch effect is due to the fact that the surface of a metallic plate cannot be an equipotential as it is constituted by micro-crystallites with different work functions. For clean metallic surfaces studied by the techniques of surface physics, the resulting “voltage roughness” is correlated to the “topography roughness” [92]. For surfaces exposed to air, the situation is changed due to the unavoidable contamination by adsorbents. The latter is known to spread out the electrostatic patches, enlarge correlation lengths and reduce voltage dispersions [93].

The pressure due to electrostatic patches between two planes can be computed exactly by solving the Poisson equation [86]. Its evaluation only depends on the spectra describing the correlations of the patch voltages or, equivalently on the associated noise spectra. In most analysis of the patch pressure devoted up to recently to Casimir experiments [56, 57], the spectrum has been assumed to be flat between two cutoffs (a minimum wavevector k_{\min} and a maximum one k_{\max}), which is a very poor representation of the patches on real surfaces. A “quasi-local” model has recently been proposed which gives a much better motivated representation of patches [91]. The model is based on a tessellation of the sample surface and a random assignment of the voltage on each patch. It produces a smooth spectrum different from the “sharp-cutoff” model used in previous analysis [56, 57]. It is worth emphasizing that the new quasi-local model shows close similarities with models used recently to explain the effect of patches on heating in ion or atom traps and cantilever damping [96, 97].

The results of the calculations of patch pressure are described in [91]. When the patch effect is estimated with the sharp-cutoff model and the same parameters as in [56, 57], a very small contribution is obtained which cannot explain the difference between experimental data and theoretical predictions using the Drude model. In contrast, when the patch effect is estimated with the quasi-local spectrum and parameters deduced from the grain sizes as in [56, 57], an unexpected result is obtained: the calculated patch pressure is now larger than the residuals (difference between experimental data and theoretical predictions using the Drude model). This means that patches are an important systematic effect for Casimir force measurements and that the patch spectrum should ideally be characterized independently of this measurement.

As the computed patch pressure is obviously model dependent, it seems natural to try to find a model between the two cases presented above which would reproduce at least qualitatively the residuals. This question has been addressed in [91] by varying the parameters of the better motivated quasi-local model. It was found that the output of the model depends mainly on two parameters, namely the size of largest patches $\ell_{\text{patch}}^{\max}$ and the rms voltage dispersion V_{rms} . A best-fit on these two parameters produces an agreement between the residuals and the patch pressure. The best-fit values for the parameters $\ell_{\text{patch}}^{\max}$ and V_{rms} are quite different from those which would be obtained by identifying patch sizes to crystallite sizes. However, as $\ell_{\text{patch}}^{\max}$ is larger than the maximum grain size $\sim 300\text{nm}$, and V_{rms} smaller than the rms voltage $\sim 81\text{mV}$ which would be associated with the random work functions of micro-crystallites, these values are reasonable for contaminated surfaces [93].

These results mean that the difference between IUPUI experimental data and theoretical predictions can be reproduced at least qualitatively by the quasi-local model for electrostatic patches. Let us stress at this point that this is the result of a fit, with the parameters of the patch model not measured independently (the same criticism holds as well for previous analysis of the data). A better characterization of the patches is a crucial condition to reaching firmer conclusions. The patch distributions can be measured with the dedicated technique of Kelvin probe force microscopy (KPFM) which is able to achieve the necessary size and voltage resolutions [94, 95]. In addition, the study of cold atoms and cold ions trapped in the vicinity of metallic surfaces [81, 96, 97] or the role of patch effects in other precision measurements [83–85] are other ways for accessing information of interest for our problem.

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